2019 **Lec 7**

2019年4月25日 13:53

Maximum matching.

Algo: maximal matching.

Thm: $\underline{\text{Algo} \geq \frac{1}{2} \cdot \text{OPT}}$ 　　　 $\text{Algo} \geq \frac{2}{n} \text{OPT}$ ←

OPT $= k$:

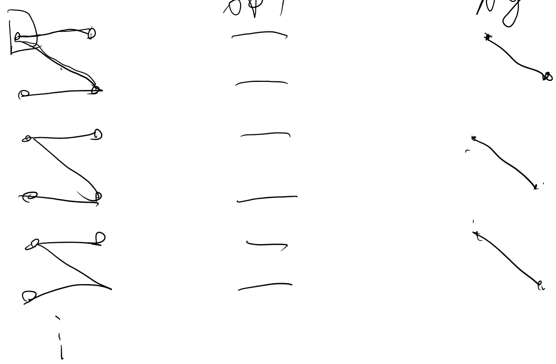
(u_i, v_i) 　 (u_j, v_j)

if Algo $< \frac{k}{2}$ 　 ×

Algo $\geq \frac{k}{2}$

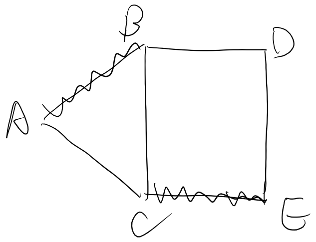
Approximation ratio $= \frac{1}{2}$. (2)

tight example: 　 OPT 　 Algo



Analysis is tight.

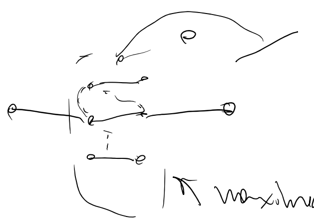
Vertex Cover.



$\{AB, CE\}$ ← maximal matching

$\{A, B, C, E\}$ ← vertex cover

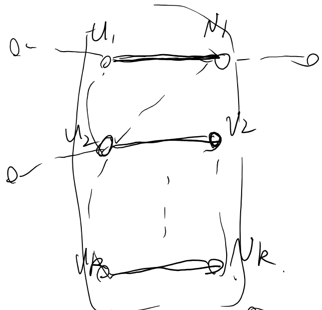
1°. why is it a vertex cover? ←



↑ maximal matching.

2°. Approximation ratio

$1 \leq \frac{\text{Algo}}{\text{OPT}} \leq ???$ 　 Algo $\geq$ OPT

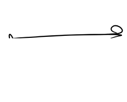


Algo $= 2k$

↑ maximal matching

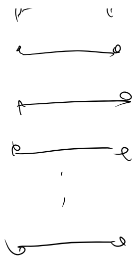
OPT $\geq k$.

$\frac{\text{Algo}}{\text{OPT}} \leq 2$.



← tight example.

$\frac{\ }{OPT} \le 2.$



← tight example.

\ maximal matching

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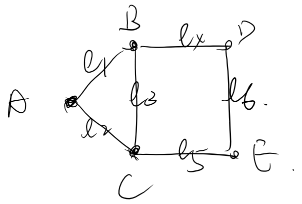
set cover.

$\{1, \cdots, 5\}$

$S_1 = \{1, 2, 3\}$ $S_2 = \{2, 4\}$ $S_3 = \{5\}$ $S_4 = \{3, 5\}$

$S_5 = \{4, 5\}$

optimal set cover: $\{S_1, S_5\}$ $= 2.$



universal set $\{e_1, \cdots, e_6\}$.

A : $\{e_1, e_2\}$ ←

B : $\{e_1, e_3, e_4\}$ ←

C : $\{e_2, e_3, e_5\}$ ←

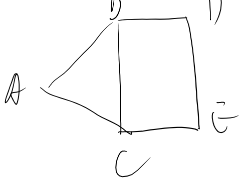
D : $\{e_4, e_6\}$ ←

E : $\{e_5, e_6\}$ ←

A, B, E

vertex cover → set cover : frequency $= 2$.

dominating set problem: vertex cover vertex.



A $\{A, B, C\}$

B : $\{A, B, C, D\}$

C : $\{A, B, C, E\}$

⋮

edge cover: edge cover vertex.

Greedy Algorithm.

1°. 0 0 0 1 1

$\{1\}$ $\{2\}$ — — $\{n\}$ $\{1, 2, \cdots, n\}$

2° choose S st $\min\{ w(S) \mid S \text{ covers at least one new element} \}$.

1	1		1	$1+\varepsilon$
$\{1\}$	$\{2\}$	— —	$\{n\}$	$\{1, \dots, n\}$

Algo $= n$

OPT $= 1+\varepsilon$.

3°. for element u_i, choose S_i st $\min\{ w(S_i) \mid u_i \in S_i \}$.

$S_1 \cup \dots \cup S_n$.

4° choose S st $\min \left\{ \dfrac{w(S)}{|\{S - \text{covered element}\}|} \right\}$.

Algo. $S_1, S_2, \dots, S_k$

$w(S_1) + w(S_2) \dots + w(S_k)$ vs $OPT = \{T_1, T_2, \dots, T_l\}$

$w(T_1) + \dots + w(T_l)$

$$\frac{w(S_j)}{|S_j - S_1 \cup \dots \cup S_{j-1}|} \le \frac{w(S)}{|S - S_1 \cup \dots \cup S_{j-1}|} \quad \forall S, \quad \forall j = \{1, \dots, k\}$$

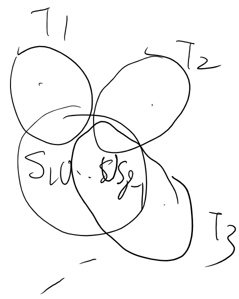
$$\le \frac{w(T_i)}{|T_i - S_1 \cup \dots \cup S_{j-1}|} \quad \forall i.$$

$\min\{\frac{1}{3}, \frac{y}{2}\} = \min\{\frac{1}{3}, \frac{2/3}{2}\}$

$\le \frac{1+y}{3+2}$ $= \frac{1+y}{3+2}$.

$$\le \frac{\sum_i w(T_i)}{\sum_i |T_i - S_1 \cup \dots \cup S_{j-1}|} = \frac{OPT}{\sum_i |T_i - S_1 \cup \dots \cup S_{j-1}|}$$

$$\le \frac{OPT}{n - |S_1 \cup \dots \cup S_{j-1}|}.$$



$$\frac{w(S_j)}{|S_j - (S_1 \cup \dots \cup S_{j-1})|} \le \frac{OPT}{n - |S_1 \cup \dots \cup S_{j-1}|} \quad \forall j.$$

$$\underline{Algo} = \sum_{\gamma} w(S_\gamma) \le \underline{OPT} \underbrace{\sum_{\gamma} \frac{1}{n - |S_1 \cup \cdots \cup S_{\gamma-1}|} \times |S_\gamma - (S_1 \cup \cdots \cup S_{\gamma-1})|}$$

$$\le O(\ln n) \cdot OPT$$

$$\frac{1}{n} \times |S_1| + \frac{1}{n-|S_1|} \times |S_2 - S_1| + \frac{1}{n - |S_1 \cup S_2|} \times |S_3 - (S_1 \cup S_2)| + \cdots$$

$$= \underbrace{\frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}}_{|S_1|} + \frac{1}{n-|S_1|} + \cdots + \frac{1}{n-|S_1|} + \frac{1}{n-|S_1 \cup S_2|} + \cdots$$

$|S_1| + 1$ 项　　$|S_2 - S_1|$　　$|S_1 \cup S_2| + 1$ 项

$$\le \underbrace{\frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{n-|S_1|+1}}_{|S_1| 项} + \frac{1}{n-|S_1|} + \cdots$$

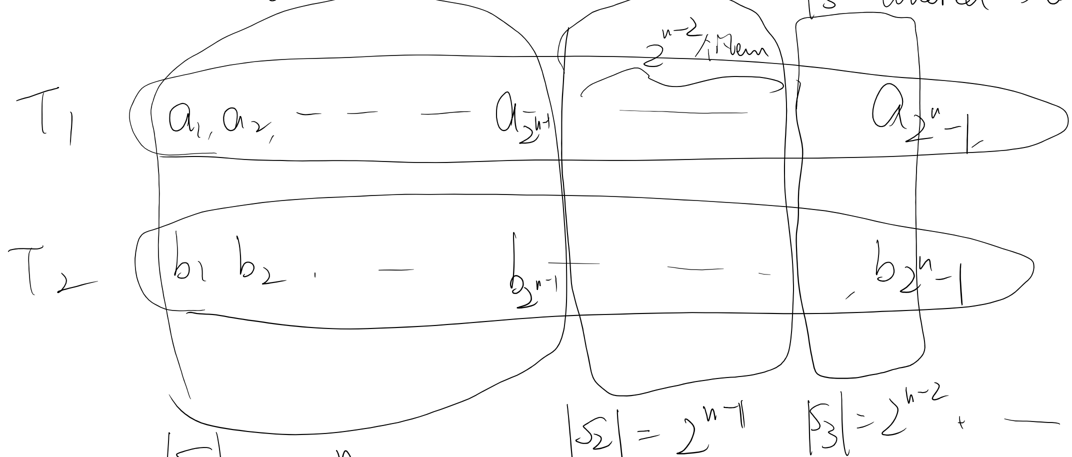
$$\le \underline{\sum_{i=1}^{n} \frac{1}{i}} \approx \underline{\ln n}$$

$\boxed{\text{tight?} \quad \Theta(\log n)}$

$$\min \frac{1}{|S - \text{covered element}|}$$

weight = 1

max $|S - \text{covered element}|$



OPT = 2 ; $\{T_1, T_2\}$.

Algo = $\underline{n}$. $\{S_1, S_2, \ldots S_n\}$

$|S_1| = 2^n$.　$|S_2| = 2^{n-1}$　$|S_3| = 2^{n-2}$ + ...

$\underline{2^{n-1} - 1}$

$N = |\text{element}| \approx \underline{2^{n+1} - 2}$

$\frac{Algo}{OPT} \cdot \Theta(\log N)$

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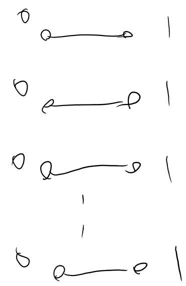
set cover　Laying Technique $\rightarrow$ approx. ratio = f.

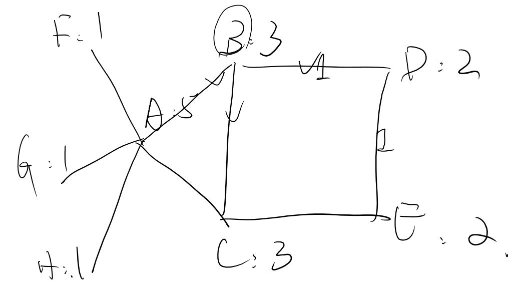
set cover   Laying Technique $\longrightarrow$ approx. ratio $= f$.

(weighted) vertex cover. $f = 2$.



$OPT = 0$

$algo = k$

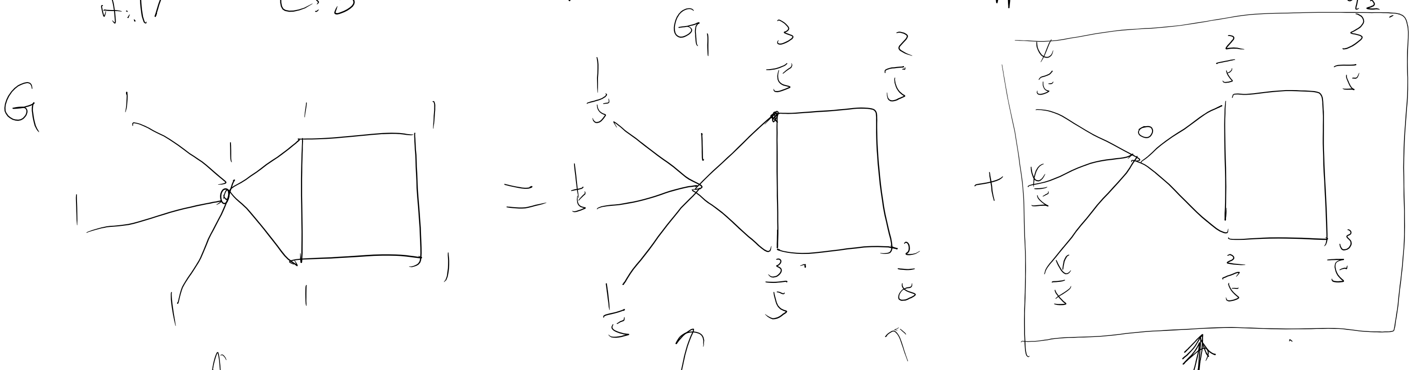


degree-weighted graph

{choose all vertices}

↑

2 - Approximation ratio.



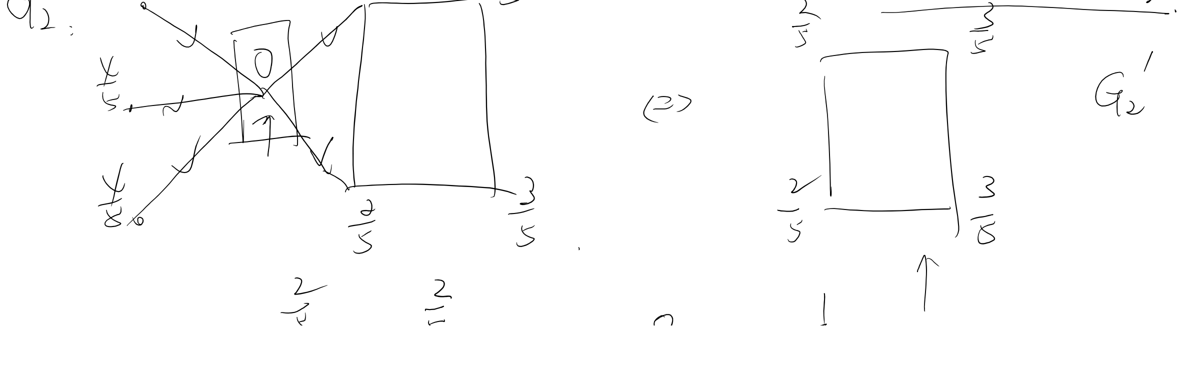
degree-weighted graph   residue graph.

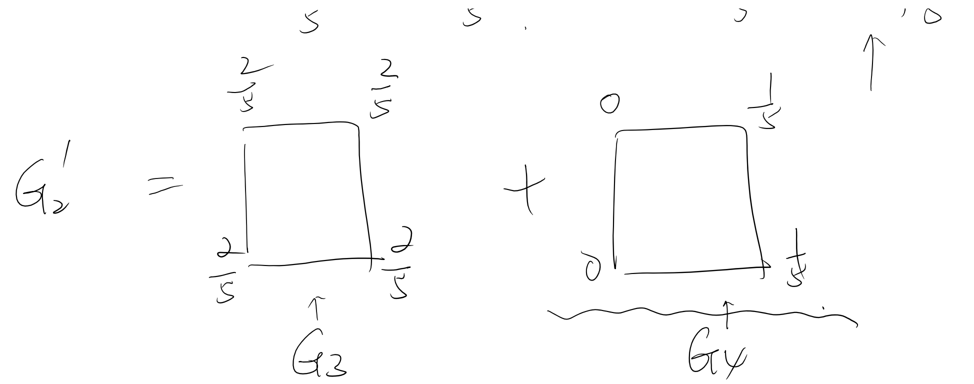
if we can find vertex cover S, s.t. $w_{G_2}(S) \le 2 \cdot OPT(G_2)$

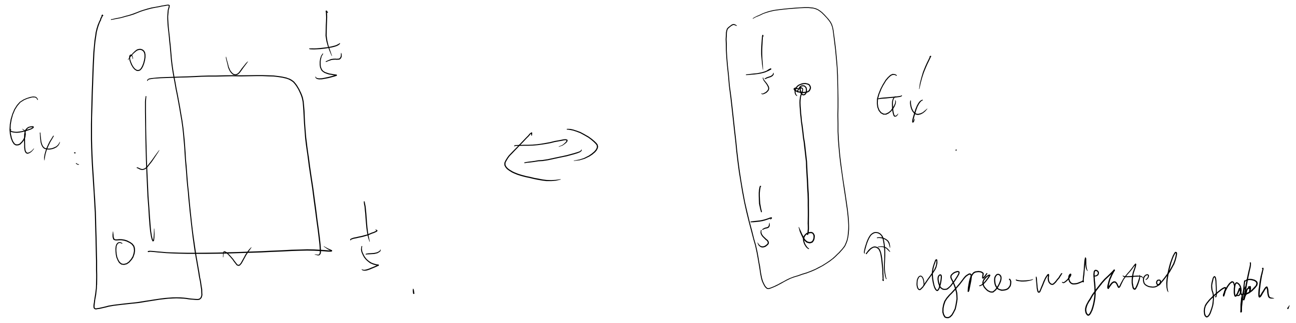
G_1 is degree-weighted graph $\Rightarrow w_{G_1}(S) \le 2 \cdot OPT(G_1)$

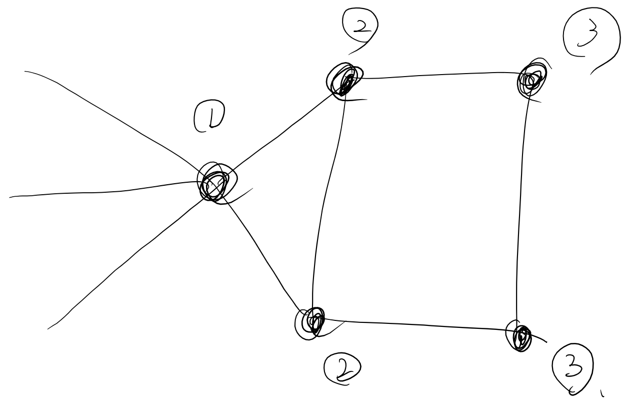
$$w_G(S) = w_{G_1}(S) + w_{G_2}(S) \le 2\left(OPT(G_1) + OPT(G_2)\right)$$

$$\le 2 \cdot OPT(G)$$

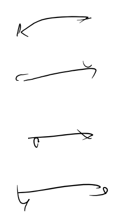








tight example.



frequency $= k$ $\longrightarrow$ Approximation ratio $= k$.

$\{e_1, e_2, ---, e_n\}$.

c

subset S_1 — vertex v_1 $= \{e_1, e_2, e_3\}$ $\Rightarrow 3$ $3 \times C$

$S_2 \sim V_2$ $= \{e_2, e_4, e_6\}$ $\Rightarrow 3$ $3 \times C$

$\{e_2, e_3, e_6, e_7\}$ $4 \times C$

e_1